Chalkboard #6 – Appendix – Equivalence Classes of Primes and Composites

Some Finite Sequence Lengths d, predicted for sequences of a divisor D of Per(d) =mod (S1(d)-SN1(d))

d	D|Per(d)	d	D|Per(d)	d	D|Per(d)	d	D|Per(d)
1	NP	11	23	21	43	31	5953
2	NP	12	35	22	23	32	97
3	NP	13	27	23	599	33	10649
4	5	14	64	24	5	34	137
5	NP	15	61	25	1151	35	2381
6	7	16	17	26	53	36	703
7	8	17	137	27	109	37	223
8	(5)	18	133	28	512	38	229
9	19	19	229	29	3451	39	79
10	11	20	275	30	61	40	281

NP = not predicted to occur

Example: A sequence of length d = 39 is predicted mod (S1(39)-SN1(39)) = mod (57591)
= mod(79*3*9*3*9) indicates a divsor D| mod(79) or D|mod(27) may have a finite sequence length of 39.
One such sequence begins (q,r,s) = (56,14,43) mod(79) and (9,0,6) mod(27)

The above table applies for prime and composite numbers.   From the formula introduced in Chalkboard #2 to calculate P(n) from prime divisors:

$P(m) = l.c.m.[\ p1^{a1-1} * p2^{a2-1} * p3^{a3-1} * \ldots p(i)^{ai-1} * P(p1) * P(p2) * P(p3)..*P(p(i))]$

Consider an example d = 18,

$P(18) = P(2*3^2) = 3*P(2)*P(3) = 3*7*13 = 273$. Then for mod 18 there are several potential equivalence classes of sequence length 273, 13, 7, (3*13 = 39), (7*13 = 91), (7*3=21). However to be in an equivalence class Per(m) must divide m.

Conjecture 1: The sequence length of the prime divisors of m are always an equivalence classes c(m) of mod(d)

e.g. P(2) = 7, and P(3) =13

Conjecture 2: If m|Per(m) or a divisor of m|Per(m) then c(m)is a equivalence class of mod(d)

C(m)	Per(m)	18|Per(m) or P(p1),.. P(pn)?
1	1	Yes always
3	1	No
7	P(2)	Yes
13	P(3)	Yes
21	344	No
39	57591 (9|18 and 9|57591)	Yes
91	= 0 mod 18	Yes
273	= 0 mod 18	Yes

Then $d^3 = \Sigma$ c(m)*Num(m) where Num is the number of elements (orbits) in the class c(m).

18^3 = 1+7*Num(7) + 13*Num(13) + 39*Num(39)+ 91*Num(91) + 273*Num(273).

Currently it is difficult to find Num(m) except by brute calculation since there are multiple solutions to the above equation. For example a possible solution is:
Num(7) = 1, Num(13) = 2, Num(39) = 18, Num(91) = 2, Num(273) = 18. This has not been proven but the sum is 5832 as required.

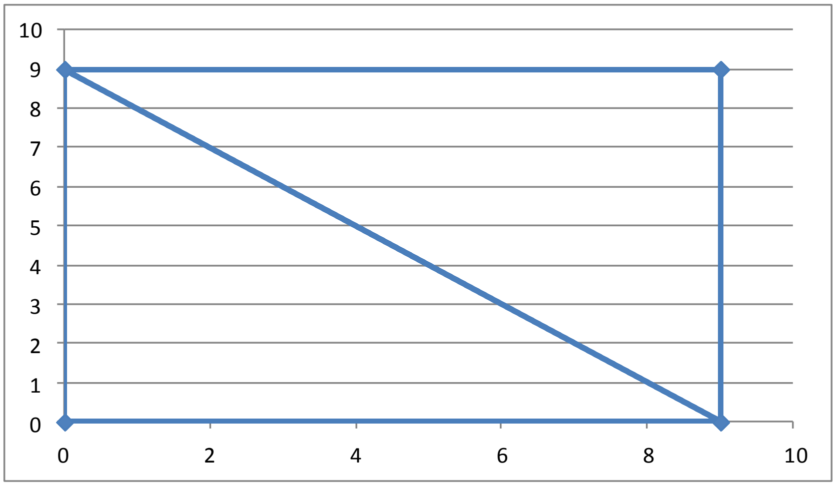
There appears a sequence of Num(m) if we look at multiples of 3 {6,9,12,18} where P(6)=91, P(9) = 39, P(12) = 182) and P(18) = 273:

1+ 1*7+2*13+2*91 = 216 = 6^3
1 +2*13+18*39 = 729 = 9^3
1 +1*7+2*13+2*91+8*182 = 1728 = 12^3
1 +1*7+2*13+18*39+2*91+18*273 = 5832 = 18^3

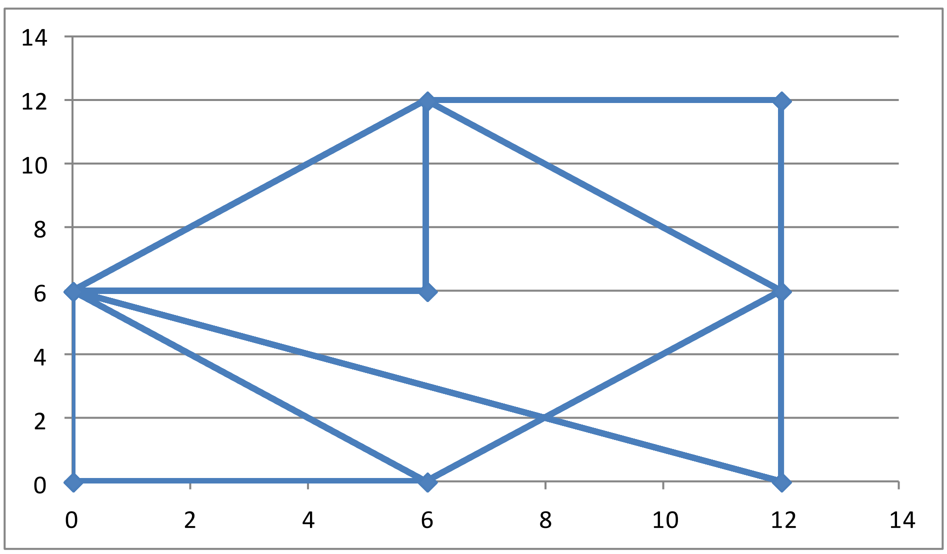
Once Num(m) is determined it is used for higher values of d.

Cycle Graphs: A visual aid to find cycle lengths can be helpful in finding Num(m). Using the example above for d = 18 a cycle graph for c(m) = 7 can be generate by using a two dimensional graph of the pairs in the sequence {a,b,c,d,e,f,…} = {(a,b),(c,d),(e,f),..}

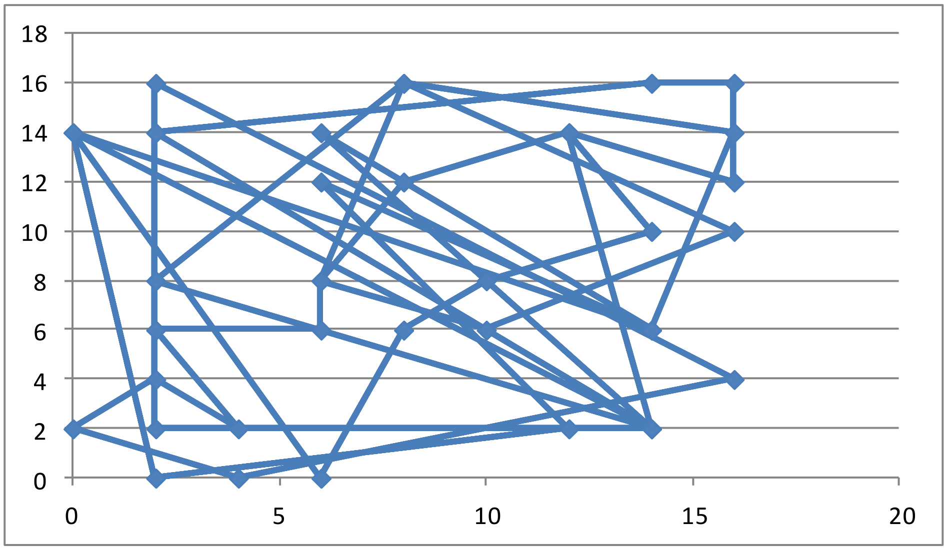
Cycle graph c(m) = 7 for mod 18 (q,r,s) = 9,9,9



Cycle graph c(m) = 13 for mod 18 (q,r,s) = 6,0,6



Cycle graph c(m) = 39 for mod 18 (q,r,s) = 6,12,2



Future Chalkboards will discuss some limitations on predicting equivalence or residue systems modulo p for linear recurrence sequences.

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