Chalkboard #2 Finite Sequences

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Finite sequences are periodic sequences. Rules are used to define these finite sequences.

A musical example is given as follows:

A song is being composed using only 2 notes (C and D). After 2 notes are played the **4th** note is played based on the following rules:

If CC played then C, If DD played then C , if DC or CD played then D.

Starting with the 3 notes DCC the song will appear as the following sequence of notes:

DCCDCDDDCCDCDDDC

Notice that after 7 notes the sequence repeats ad infinitum.

Note: *If the second note was E instead of D in the example above the sequence above would end as Beethoven's Fifth Symphony begins!*

Given the infinite Perrin Sequence 3,0,2,3,2,5,5,7,10,12,17,22,29,… convert each number to modulo 2 which will give a series of 0's and 1's:

1001011100101….

The rules of addition modulo 2 are

1+1 =0, 0+0 =0, and 1+0 = 0+1

Note that these rules are equivalent to the rules for C and D above!

We can conjecture that for any modulo n (abbreviated mod(m)) there will be a finite sequence of m numbers having a sequence which repeats ad infinitum.

The following Table shows the calculated period of the sequences P(m) for the first 37 integers mod(m).

m	P(m)	m	P(m)	m	P(m)
2	7	14	336	26	1281
3	13	15	312	27	117
4	14	16	56	28	336
5	24	17	288	29	871
6	91	18	273	30	2184
7	48	19	180	31	993
8	28	20	168	32	112
9	39	21	624	33	1560
10	168	22	840	34	2016

11	120	23	22	35	48
12	182	24	364	36	546
13	183	25	120	37	1368

Unlike the Perrin sequence which is calculated from S1(m-2) and S1(m-3) no obvious pattern is observed for the periodic sequence. Some periods are short (m = 23) and others are long (m=30).

Since 0 mod(m) is repeated every P(m) Perrin numbers, because S1(1)=0, the property of the P(m) +1 number in the perrin sequence is known. That is, m divides S1(P(m)+1) or

m|S1(P(m)+1)

As examples 3|S1(P(3)+1) = 3|S1(14) = 3|51 equivalent to 0 = 51mod(3)

11|S1(121),

29|S1(872)

This is true for all multiples of P(m)

m|S1(k*P(m)+1) where k is an integer >1

Example: 3|S1(66) where k = 5,

29|S1(2614) where k = 3

Although numbers like S1(121) are very large [> $5.9x10^{14}$], using modulo arithmetic is convenient since numbers mod(m) are never larger than m-1. This will be important in future chalkboard calculations.

Prime Decomposition

The prime decomposition theorem states that any integer can be written as a product of primes raised to a power.

$$m = p1^{a1}*p2^{a2}*p3^{a3}*\ldots p(i)^{ai}$$

where p(i) are prime numbers.

Examples:

$28 = 2^2*7^1$

66 =2*3*11

$20 = 2^2*5$

Conjecture: P(m) is calculated from the prime decomposition of integer m.

$P(m) = l.c.m.[\ p1^{a1-1} * p2^{a2-1} * p3^{a3-1} * \ldots p(i)^{ai-1} * P(p1) * P(p2) * P(p3) .. * P(p(i))]$

Where l.c.m is the lowest common multiple, p(i) are primes and P(p(i)) are the Perrin sequence lengths.

Examples:

Calculate P(28)

$P(28) = l.c.m\ [2^{2-1} * 7^{1-1} * P(2) * P(7) = l.c.m\ [2*7*48] = 336$ since 48 = 2*8*3 (2 is a common multiple and $7^0 = 1$). This agrees with the Table for P(28) above.

$P(20) = l.c.m\ [2^{2-1} * 5^{1-1} * P(2) * P(5)] = l.c.m[2*7*24] = 168$

For higher values of m P(m) can be calculated:

$P(66) = l.c.m\{2^{1-1} * 3^{1-1} * 11^{1-1} * P(2) * P(3) * P(11)] = 7*13*120 = 2^3*3*5*7*13 = 10920.$

The use of the prime decomposition theorem requires that P(m) values are calculated for m when m is a prime number. [e.g.] How can we calculate P(941) where 941 is a prime?

Next Chalkboard: Predicting Perrin sequence lengths of Primes.

RT