Appendix to [α,β] Classes of Elliptic Equations

Let:

$g(z) := z^3 - \alpha \cdot z - \beta$ with α and $\beta > 0$

Find Conditions for which the Perrin sequence has sequence length of 1 (mod1) and sequence length 2 (mod 2)

conjecture: for $\alpha > \beta$

mod1=α+β-1

mod2/mod1 = $\gamma = \alpha$-β-1
mod2 =γ*α

x = mod1
y = mod2-x

example α=23 β=7

mod1= 23+7-1 = 29

$\gamma = \alpha$-β-1 = 23-7-1 = 15

mod2= 15*29=435

Then
x=mod1=29
y=435-29=406

also, mod2 = $(\alpha-1)^2-\beta^2$

conjecture: for $\alpha<\beta$

mod1=α+β-1

mod2/mod1 = $\gamma=\beta-\alpha+1$
mod2 =$\gamma*\alpha$

x = mod1
y = mod2-x

example: α=7 β=23

mod1= 23+7-1 = 29
γ= 23-7+1= 17

mod2= 17*29=493

Then
x=mod1=29
y=493-29=464

also, mod2 = $-(\alpha-1)^2+\beta^2$

For a sequence from x^3-23*x-7 For a sequence from x^3-7*x-23

SEQUENCE LENGTH 1: mod1 = 29 for both equations: Numbers 1 thru 28 give sequences

(1,1,1,1,..), (2,2,2,2,,...) etc. mod 29

SEQUENCE LENGTH 2:

$i := 1..20$

$S11_1 := 29$

$S11_2 := 406$

$S11_3 := 29$

$S11_{i+3} := S11_i \cdot \beta + S11_{i+1} \cdot \alpha$

$\mathrm{mod}(S11_i, 435)$

29
406
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406
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406
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406
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406
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406
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406
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406
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406

$i := 1..20$

$S111_1 := 29$

$S111_2 := 464$

$S111_3 := 29$

$S111_{i+3} := S111_i \cdot \alpha + S111_{i+1} \cdot \beta$

$\mathrm{mod}(S111_i, 493)$

29
464
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464
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464
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464

Reduce solution to common factors (x,y) to get the number of orbits associated with mod2

#Orb = 1/2 * (mod2-mod1) with adjusted modulus =SUM(x,y)

(29,406) ~ (1,14) and transitive multiples (29,464) ~ (1,16)

29*14/2 = 203 orbits 29*16/2 = 232 orbits

RT