Appendix to #13: Perrin Pseudoprimes and a Binary Prime Number Sieve

From Chalkboard #13 I showed that Perrin pseudoprimes (n) are confirmed by an Euler phi (totient) transform on the Perrin number such that:

[1]   n*sigma orbit – Perrin (n) = Σ [d|n](Perrin(d)*ϕ (n/d)) with d< n

The sigma orbit = OEIS A127687 and ϕ = A000010 and d|n indicate all d dividing n.

Note that at d=1, Perrin(1) = 0

For large n, this equation is tractable by

[2] Σ [d|n](Perrin(d)*ϕ (n/d)) mod n = 0

Based on the definition of the sigma orbit and Perrin number, the LHS of [1] is defined as the difference in the number of unlabeled maximal independent sets of an n-cycle graph times n from Perrin(n) which is also identified as the number of distinct maximal independent sets of the n-cycle graph.

I have looked at the absolute value of the LHS of [1] and calculated up to n=56 (numbers available on A127687) in the table below:

First 56 Values of N*sigma_orbit – Perrin(n) and its mod 2 value

N	Perrin	sigma orbit	N*sigorb-Perrin	mod 2
1	0	0	0	0
2	2	1	0	0
3	3	1	0	0
4	2	1	2	0
5	5	1	0	0
6	5	2	7	1
7	7	1	0	0
8	10	2	6	0
9	12	2	6	0
10	17	3	13	1
11	22	2	0	0
12	29	4	19	1
13	39	3	0	0
14	51	5	19	1
15	68	6	22	0
16	90	7	22	0
17	119	7	0	0
18	158	11	40	0
19	209	11	0	0
20	277	16	43	1
21	367	19	32	0
22	486	24	42	0
23	644	28	0	0
24	853	39	83	1
25	1130	46	20	0
26	1497	60	63	1
27	1983	75	42	0
28	2627	97	89	1
29	3480	120	0	0
30	4610	159	160	0
31	6107	197	0	0
32	8090	257	134	0
33	10717	327	74	0
34	14197	422	151	1
35	18807	539	58	0
36	24914	700	286	0
37	33004	892	0	0
38	43721	1157	245	1
39	57918	1488	114	0
40	76725	1928	395	1
41	101639	2479	0	0
42	134643	3219	555	1
43	178364	4148	0	0
44	236282	5383	570	0
45	313007	6961	238	0
46	414646	9029	688	0
47	549289	11687	0	0
48	727653	15184	1179	1
49	963935	19673	42	0
50	1276942	25564	1258	0
51	1691588	33174	286	0
52	2240877	43125	1623	1
53	2968530	56010	0	0
54	3932465	72868	2407	1
55	5209407	94719	138	0
56	6900995	123283	2853	1

Since prime values of n are not composite the RHS of [1] is always 0. It is interesting to note that the value of N*sigma_orbit – Perrin(n) is either odd or even and the last column indicates this by taking each number mod 2.

A recurring binary sequence [00000100010101] of 14 digits is found that repeats 4 times in the above table. If the recurring binary sequence is carried out to N=1000, all primes are found to occur in the region of a "0".

Of particular interest are twin primes such as (example: 41,43; 191,193; 311,313; 461,463; 659,661; 821,823; 881,883) which occur between the "1s" in the binary sequence.

If the four "1s" in this sequence are carried out as multiples of 14 then the following theorem is proposed:

Theorem I:

For any number X, a prime number may occur within the numbers 14*X +6, 14*X +10, 14*X +12, and 14*X +14 (or 14*(X+1)). If a prime does not occur in this range then it will occur in the range 14*(X+a) +6, 14*(X+a) +10, 14*(X+a) +12, and 14*(X+a) +14 where a is a small integer (a can also be a negative integer). This sieve can be represented as a binary sieve of 14 digits [00000100010101] spanning the integers.

Example:

Look for primes around 300000. X=21428 and a = -1 and 0. The ranges are 299992, 299998, 300002,300004, 300006 and 300012. Two primes, 299993 and 300007 are prime in this range.

Look for primes around 3000000. X=214285 and a = 0 and 1. The ranges are 2999996, 3000000, 3000002, 3000004, 3000010, 3000014, 3000016 and 3000018. Two primes, 2999999 and 30000017 are prime in this range.

<u>Region of Perrin Pseudoprimes</u>

The region around a perrin pseudoprime is found to be 0 mod2 from the above theorem.

PPP(1) = 271441

From Theorem I, X = 19388, a = -1 and 0 and the ranges are 271432, 271438,271442, 271444, 271446. It is found that 271438<PPP(1) <271442. Primes occur outside this range of 14 at 271429 and 271451.

PPP(3) = 16532714

From Theorem I, X = 1180908, a = -1 and 0 and the ranges are 16532712, 16532718, 16532722, 16532724, 16532726. It is found that 16532712<PPP(3) <16532718. One prime occurs in this range of 14 at 16532713 and the next prime outside the range of 14 (16532726+14 = 16532740) at 16532741.

The binary prime number sieve 00000100010101 (if this proposed theorem is true) may have important implications in number theory, prime counting, and the Riemann prime number formula.

I note here that the first type 3 perrin prime is 23 (S signature). 14/23 = 0.60869.. ~ $6/\pi^2 = 1/\zeta(2)$

Where $\zeta(x)$ ***is the Riemann zeta function and*** $1/\zeta(2)$ ***is the probability that two integers picked at random are relatively prime! ---*** **Richard Turk Plymouth MA August 22, 2015**

Completing the Binary Prime Number Sieve

For each multiple of 14 there are 14 bins which may contain a prime or a Perrin pseudoprime.

Starting from the second sequence multiple (integers 15-29) the first 10 multiples fill bins with prime numbers as follows:

BIN/	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Binary bit	0	0	0	0	0	1	0	0	0	1	0	1	0	1
Sequence Multiple														
2			3		5				9					
3	1		3						9				13	
4	1				5						11			
5			3		5						11			
6	1		3						9				13	
7			3										13	
8			3		5				9		11			
9	1													
10	1				5						11		13	
= mod7	1		3		5		0		2		4		6	

Example: for integers 15-28, primes 17,19 and 23 fill bin 3, 5 and 9.

As the process is repeated it is noted that bins with binary bit 1 are not filled and bins 2,4,7(except for the prime 7), and 8 are empty even though they have binary bit 0.

Each filled column is a prime number that is a residual of mod 7. Each prime in bin 1, 3 and 5 is residual 1 mod7, 3 mod 7 and 5 mod 7. Each prime in bin 9, 11 and 13 is residual 2 mod7, 4 mod 7 and 6 mod 7.

Example: the prime 857 = 3 mod 7 and falls in bin 3.

Based on this sieve method, primes mod 7 only fill bins 1,3,5,9, 11 and 13. Composite Perrin pseudoprimes can fill any bin with a 0 bit. Eg; PPP(1) bin 9, PPP(3) bin 2.

Richard Turk August 23, 2015