Perrin Prime Distribution Theorem

To paraphrase "God created the integers": God created the integers 1,2,3,4,5,6 mod 7 in six days and on the last day he created 0 mod 7 and rested from all his work he had made.

In Chapter 13 and (1) I introduced the Euler phi transform of the Perrin sequence. Given the integer ***I*** as the number of unlabeled maximal independent sets of an N-cycle, I show that

[1] **N****I*** – Perrin (N) = Σ [d|N](Perrin(d)*ϕ (N/d)) with d< N**

Implies for Perrin primes and Perrin pseudoprimes that

[2] **Σ [d|N](Perrin(d)*ϕ (N/d)) mod N = 0**

and the LSH of [1] is zero.

A calculation of the first 56 values of ***I***, Perrin(d), and ϕ (N/d) show the LHS of [1] is either even, 0, or odd. If the value **N****I*** – Perrin (N)** is calculated mod 2, the result is a binary series of "0s" and "1s" with a periodic sequence of 14. [00000100010101].

A grid containing all natural integers can be inserted over integers arranged in rows of 14 and each row labeled 14X where X = 0,1,2….. The 2D position coordinates of each integer are (b_i, X) with b_i representing the bit or Bin (i= 1,2,3,..14)and X the row with X= 0,1,…. the row number as multiples of 14.

When the prime numbers are positioned on this grid they occupy inclusive of all primes except 2, and 7 the 6 position coordinates (1,X), (3,X), (5,X), (9,X), (11,X), and (13,X). As indicated above bit 6,10, 12,and 14 cannot contain a prime or pseudoprime. Bits 2, 4 and 8 are even and except for the prime number 7 (position (7,0) all positions (7,X)are multiples of 7.

Theorem [1]: Perrin Prime Number Distribution

*All primes N which are Perrin primes such that N|Perrin(N) have position coordinates of either (1,X), (3,X), (5,X), (9,X), (11,X), or (13,X) where 1, 3,5,9,11, and 13 are positions of the binary sequence 00000100010101 and X is an integer 0 or >0 such that N = 14*X +* b_i *where* b_i *is the bit 1,3,5,9,11 or 13. For X=0, the positions (2,0) and (7,0) are also included.*

It can easily be shown that the 6 positions above contain both primes and composites of primes since only these 6 positions contain prime numbers.

It can also be shown that these positions correspond to modulo 7 numbering system such that Bin 1 = 1 mod 7, Bin 3 = 3 mod 7, Bin 5 = 5 mod 7, Bin 9 = 2 mod 7, Bin 11 = 4 mod 7, Bin 13 = 6 mod 7.

Using a modulo 7 multiplication table it can be shown that there are 4 non equivalent ways to obtain 1 mod 7 (1x1, 2x4, 3x5, and 6x6). Similarly, there are 4,3,4,3 and 3 ways to obtain 2,3,4,5, and 6 mod 7.

I introduce the bin transform function β which converts the above numbers to the appropriate set of Bin multiples.

[3] β: n mod 7 ⟶ $\{(b1_i \text{ X } b2_i) \mid (b1_i, b2_i) = 1,3,5,9,11,13\}$

These bins are the only bins that contain the prime divisors of a number N, position (b_i, X). These divisors can be in the same bin as N or in a bin allowed by the bin transform function.

Given a number that is in one of the 6 bin positions (1,X), (3,X), (5,X), (9,X), (11,X), and (13,X) the method below can be used to show for N= C*D with either C and D prime , or C or D prime, whether N is a composite number with at least one number not equal to 1 which is a prime divisor of N.

Theorem 2: Prime Divisor Function

*Given N= C*D and position coordinates for N; (b_i, X) with b_i = n mod 7 and a given*

bin transform β: n mod 7⟶ {(b1 X b2)} the following non-linear function in x, y holds:

[4] $f(x,y,N) = 14xy + b2*x + b1*y - Z(N) = 0$

*Where Z(N) = integer = (N-b1*b2)/14 and x, y are integer solutions of [4] with partial derivatives*

$C= (dF/dy)_{x,N} = 14*x + b1$ *and* $D = (dF/dx)_{y,N} = 14*y + b2$

Let d(N) be the number of prime divisors of N. Then

$d(N) = \Sigma_{(\text{all } \beta:\text{n mod7})} (\#y_i \mid f(x_i,y_i,N) = 0)$ where the sum is over all bin multiples (b1xb2).

[Note: bin multiple(1xb2) includes the divisor 1 and N.]

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Example: N = 41633 has position coordinates (11,2973) so 11= 4 mod7

β: 4 mod 7 ⟶ {(3 X 13)}

F(x,y) = 0 implies $14xy + 13x + 3y = (41633-(3*13))/14 = Z = 2971$

Rearrange equation [4] to calculate y from x:

[4a] $y = (Z-b2*x)/(14*x+b1)$

Then for x= 1 y = 174 or

*C= 14*1+3 =17*

*D= 14*174+13 = 2449*

*Since C is prime we know that N = 41633 is composite. The procedure if needed for prime decomposition of N is repeated and shown that N = 1*17*31*79. Position coordinates for the prime divisors are (1,1), (3,1),(3,2) (9,5)and (11,2973) so d(N) = 5*

Note that for bin 11 there are three other potential bin multipliers; (1X11), (9X9), (5X5) as well as (3X13). It can be shown that there are no other integer solutions except for the trivial N= 1*N obtained from the multiplier (1x11).

It can easily be shown that the next vertical bin 11 number (11,2973+1) = 41647 is prime since no solution except the trivial solution occurs for the bin multipliers. Addition of 14 to a number in a given bin moves the position to the next number below in the column. In this way the behavior of the solution of equation [4] determines the vertical prime gaps.

Corollary to Theorem 2

For a given value of N the maximum allowed value of x (x_{max}) *< Z/(14+b1).*

Although (x_{max})$>N^{1/2}$ *the bin multiplier can be reversed (b1xb2) ➔ (b2xb1),*

then (x_{max}) *< [[Z/(14+b1)]-b2*b1]]/(14+b2).*

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