Addendum to Chapter 14: Gamma,Beta functions and Integral expansion of orb1σ

1. Example Calculation using factorials, N = 37

$N := 37$ $k := 5$

$i := 1..k$

$A_0 := \frac{N-3}{2}$ $B_0 := 1$

$A_i := A_{i-1} - 3$ $B_i := B_{i-1} + 2$

$i := 0..k$

$$orb\sigma_i := \frac{(A_i + B_i - 1)!}{A_i! \cdot B_i!}$$

$A_i =$

17
14
11
8
5
2

$B_i =$

1
3
5
7
9
11

$orb\sigma_i =$

1
40
273
429
143
6

$$orb1\sigma := \sum_i orb\sigma_i$$

$orb1\sigma = 892$

$Perrin := N \cdot orb1\sigma$

$Perrin = 33004$

orb1σ divides Perrin(N) N times only if N is Prime

The calculations below produce the same result as Example 1

2. Example Calculation using the Gamma function,   N = 37

$$\mathrm{orb11}\sigma_i := \frac{\Gamma\left(A_i + B_i, 0\right)}{\Gamma\left(A_i + 1, 0\right)\cdot\Gamma\left(B_i + 1, 0\right)}$$

$\mathrm{orb11}\sigma_i =$

1
40
273
429
143
6

$$\mathrm{orb11}\sigma := \sum_i \mathrm{orb11}\sigma_i$$

$\mathrm{orb11}\sigma = 892$

3. Example Calculation using the Beta function N = 37

The Beta function is defined as:

$$\mathrm{Beta}_i := \frac{\Gamma\left(A_i, 0\right)}{\Gamma\left(A_i + B_i, 0\right)}\cdot\Gamma\left(B_i, 0\right)$$

$$\mathrm{orb111}\sigma_i := \frac{1}{A_i\cdot B_i\cdot \mathrm{Beta}_i}$$

$\mathrm{orb111}\sigma_i =$

1
40
273
429
143
6

$$\mathrm{orb111}\sigma := \sum_i \mathrm{orb111}\sigma_i$$

$\mathrm{orb111}\sigma = 892$

3. Example Calculation using the Integral form of the Beta function N = 37

$m_i := A_i$ $n_i := B_i$

$$I_i := 2 \cdot \int_0^{\frac{\pi}{2}} \cos(\theta)^{2 \cdot m_i + 1} \cdot \sin(\theta)^{2 \cdot n_i + 1} \, d\theta \cdot (m_i + n_i + 1) \cdot (m_i + n_i)$$

$$orb1111\sigma_i := (I_i)^{-1}$$

$orb1111\sigma_i =$

1
40
273
429
143
6

$$orb1111\sigma := \sum_i orb1111\sigma_i$$

$orb1111\sigma = 892$

The last example illustrates an integral relation between prime numbers and trigonometric functions!!

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