Appendix to the Perrin Prime Number Distribution

In Chapter 14 discussing the Perrin Prime number distribution, it was shown that all prime numbers are found in bit 1,3,5,9,11, and 13 of a 14 by 14X (x=1,2,3…) grid where prime numbers 2 and 7 are only found when X=0. Since the prime decomposition of the composite number N = $p1^a*p2^b*p3^c*$…also found in these 'bins' are products of prime numbers (p1,p2,p3..) in these bins the prime divisor function can be used to find a divisor of a given number N such that p|N by choosing the appropriate bin transform.

Although an algorithm is given to find divisors of composite numbers in bins 1,3,5,9,11 and 13 it requires a search for the solution to a function F(x,y,N) = 0. For large numbers, a computer search algorithm can be used to prove whether a given number is prime or composite.

It may be (relatively) easy to determine a prime based on the number of its maximal independent sets (MIS). Both Perrin primes and Perrin Pseudoprimes are found when

[1] n*sigma orbit – Perrin (n) = Σ [d|n](Perrin(d)*ϕ (n/d)) with d< n

The RHS of equation [1] is identically zero when n is prime. If n is a Perrin pseudoprime then the RHS equals zero mod(n) since Perrin(n) mod (n) =0 by definition of a pseudoprime.

It was shown by R. Bisdorff and JL. Marichal, *Counting non-isomorphic maximal independent sets of the n-cycle graph*, J. of Integer Sequences, Vol. 11 (2008) that the sigma orbit which is the Euler totient transform of the Perrin sequence can be further transformed to the sigma(1) orbit using a Dirichlet convolution product [2]:

[2] (f*g)(n) = $\Sigma_{[d|n]}$**f(d)*g (n/d)**

where f(d) = sigma orbit(n) = $orb^\sigma(n)$

and g(n/d) = μ(n) = mobius function OEIS A008683

[2a] **$orb1^\sigma(n) = \Sigma_{[d|n]} orb^\sigma(d) * \mu(n/d)$**

Theorem A1

As shown by Bisdorf and Marichal for all Perrin Primes: ***$orb^\sigma(n) = orb1^\sigma(n)$*** *d=1,n where for all other numbers not prime,* ***$orb^\sigma(n) > orb1^\sigma(n)$***

This is a powerful theorem which can be used to distinguish a prime from a composite number. It is noted that the significance of ***$orb1^\sigma(n)$*** is that for any d|n it gives the number of unlabeled MISs of the n-cycle having n/d isomorphic representations. Furthermore, from [2a] it shows the equality in Theorem A1 only if the divisors of n are n and 1.

Although $orb1^{\sigma}(n)$ could be calculated from the sigma orbits and mobius function $\mu(n)$ it is more convenient to find an alternate method of calculation.

Combinatorics of the Sigma(1) orbit

The principle of MISs is that all numbers are the sum of a number of 2s and a number of 3s. The function **$orb1^{\sigma}(n)$** only gives the number of unique symmetric combination of 2s and 3s. An example is given for N=37 in the table below where a factorial combination finds the number of (A,B) symmetric forms:

Number of 2s	Number of 3s				
A	B	2A +3B	A+B	(A+B-1)!/A!B!	**$orb1^{\sigma}(37)$**
17	1	37	18	**1**	___
14	3	37	17	**40**	___
11	5	37	16	**273**	___
8	7	37	15	**429**	___
5	9	37	14	**143**	___
2	11	37	13	**6**	___
			Sum	**892**	**892** (1)

1. Calculated from Perrin(37)/37 = 33004/37 = 892

The distinction between primes and composites is the ability to obtain integers in the factorial calculations (column 5). It is observed that integer calculation is maintained in factorials of the numbers in column 1: A0 = (N-3)/2, A1 = A0-3, A2 = A1-3, etc. and column 2: B0 = 1, B1 = B+2, B2 = B1+2,…

For composite numbers, there is always one row in the calculation in which either the numerator (A+B-1)!<A!B! or a divisor B! does not divide (A+B-1)!/A!. This can easily be demonstrated if 3|N or 5|N but in general it occurs if B|A or A|(A+B-1) in some row. It is simple to calculate a table for any number found in Bin 1,3,5,9,11 or 13 to determine if it is prime. It involves no more than N-2/3 rows or an algorithm to find conditions in which B|A or A|(A+B-1). Many times, if B|A then B is also a divisor of N.

If the last row gives A = 0 then the factorial is less than 0 and N is a composite. Equation [3] applies for all primes:

[3] $$orb1^{\sigma}(n) = orb^{\sigma}(n)^* = \sum_{(\text{all } A_i = A_{(i-1)}-3,\ B_i = B_{(i-1)}+1)} (A_i+B_i-1)!/A_i!B_i!$$

Richard Turk Plymouth MA September 7, 2015