Convolution Recurrences from Inter Sequence Polynomials (CRISP)

ISP's (Inter Sequence Polynomials) are used to find the Recurrence sequences for Convolution sequences of Third order and Second order Parent Sequences. The Element sequence has a recurrence similar to the Parent but the leading sequence terms are z-derivatives of the ISP. The convolution sequences do not have a recurrence similar to the Parent or Element. It was possible by using higher order ISP terms to obtain these recurrence coefficients for the sequences of the second and third derivatives of the ISP in terms of c0, c1 and c2. These sequences are convolutions of the Element with itself and the convolution of the Element with the second derivative . The recurrence coefficients for a third order parent sequence are found as CFS and FCS for its second and third ISP derivatives in the program below, respectively. The recurrence coefficients for a second order parent sequence are found as CFS2 and TCS for its second and third ISP derivatives, respectively. The order of the convolution sequence is found to be the sum of the orders of its component sequences. For example a third order Element sequence when self convoluted results in a sixth order recurrence. A sixth order when convoluted with a third order results in a ninth order recurrence.

Linear Parent recurrences result in linear higher order convolution sequences. The Mathematica program below computes these convolution sequences from the ISP and its z derivatives.
The ISPs are given as fifth order polynomials, but the program is applicable for third and second order polynomials only. The examples given are for the Tribonacci numbers
WH = (0, 0, 1, 1, 1); OEIS A000073 and Jacobsthal numbers WH2 = (0, 0, 0, 2, 1); OEIS A001045. The program can be applied to any parent sequence given coefficients c0, c1 and c2. The ISPs and their z-derivatives provide the appropriate leading terms for these sequences.

R. Turk November 15, 2019

Third order Parent Sequence for x^3 -c2*x^2-c1*x-c0 = 0 WH = (0, 0, c0, c1, c2)

```
In[457]:= WH = {0, 0, 1, 1, 1}
Out[457]= {0, 0, 1, 1, 1}

In[458]:= x = WH[[3]]
Out[458]= 1

In[459]:= y = WH[[4]]
Out[459]= 1

In[460]:= z = WH[[5]]
Out[460]= 1
```

The Parent Sequence (ISPs)

```
In[461]:= ISP1w[v_, w_, x_, y_, z_] := z

In[462]:= ISP2w[v_, w_, x_, y_, z_] := 2 y + z^2
          ISP3w[v_, w_, x_, y_, z_] := 3 x + 3 y z + z^3

In[464]:= RecurrenceTable[
           {a[n] == WH[[5]] * a[n - 1] + WH[[4]] * a[n - 2] + WH[[3]] * a[n - 3], a[0] == ISP1w[0, 0, x, y, z],
            a[1] == ISP2w[0, 0, x, y, z], a[2] == ISP3w[0, 0, x, y, z]}, a, {n, 0, 22}]

Out[464]= {1, 3, 7, 11, 21, 39, 71, 131, 241, 443, 815, 1499, 2757, 5071,
           9327, 17155, 31553, 58035, 106743, 196331, 361109, 664183, 1221623}
```

The Element Sequence (First derivative of the ISP)

```
In[465]:= IDz1w[v_, w_, x_, y_, z_] := 1

In[466]:= IDz2w[v_, w_, x_, y_, z_] := 2 z

In[467]:= FD3 = RecurrenceTable[{a[n] == WH[[5]] * a[n - 1] + WH[[4]] * a[n - 2] + WH[[3]] * a[n - 3],
             a[0] == 0, a[1] == IDz1w[0, 0, x, y, z], a[2] == IDz2w[0, 0, x, y, z] / 2}, a, {n, 0, 22}]

Out[467]= {0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927,
           1705, 3136, 5768, 10609, 19513, 35890, 66012, 121415, 223317}
```

Second derivative- Convolution of Element sequence with itself

```
In[468]:= CFS = {2 * z, 2 * y - z^2, 2 * (x - z * y), -2 x * z - y^2, -2 * x * y, -x^2}

Out[468]= {2, 1, 0, -3, -2, -1}

In[469]:= ISDz2w[v_, w_, x_, y_, z_] := 2

In[470]:= ISDz3w[v_, w_, x_, y_, z_] := 6 z

In[471]:= ISDz4w[v_, w_, x_, y_, z_] := 8 y + 12 z^2

In[472]:= ISDz5w[v_, w_, x_, y_, z_] := 10 x + 20 y z + 8 z^3 + z (10 y + 12 z^2)

In[473]:= ISDz6w[v_, w_, x_, y_, z_] := 12 w + 18 y^2 + 36 x z + 72 y z^2 + 30 z^4

In[474]:= ISDz7w[v_, w_, x_, y_, z_] := 14 v + 42 x y + 42 w z + 84 y^2 z + 84 x z^2 + 140 y z^3 + 42 z^5

In[475]:= S3D = RecurrenceTable[{a[n] == CFS[[1]] * a[n - 1] + CFS[[2]] * a[n - 2] +
                 CFS[[3]] * a[n - 3] + CFS[[4]] * a[n - 4] + CFS[[5]] * a[n - 5] + CFS[[6]] * a[n - 6],
             a[0] == ISDz2w[0, 0, x, y, z] / 2, a[1] == ISDz3w[0, 0, x, y, z] / 3,
             a[2] == ISDz4w[0, 0, x, y, z] / 4, a[3] == ISDz5w[0, 0, x, y, z] / 5,
             a[4] == ISDz6w[0, 0, x, y, z] / 6, a[5] == ISDz7w[0, 0, x, y, z] / 7}, a, {n, 0, 22}]

Out[475]= {1, 2, 5, 12, 26, 56, 118, 244, 499, 1010, 2027, 4040, 8004, 15776,
           30956, 60504, 117845, 228818, 443057, 855732, 1649022, 3171128, 6086626}
```

Third Derivative -Convolution of second derivative with element sequence

```
In[476]:= FCS = {3 z, 3 (y - z^2), z^3 - 6 z * y + 3 x, -3 (y^2 + 2 x z - y z^2),
            -3 (2 x y - y^2 z - x z^2), -3 x^2 + y^3 + 6 x y z, 3 x^2 * z + 3 y^2 * x, 3 * y * x^2, x^3}

Out[476]= {3, 0, -2, -6, 0, 4, 6, 3, 1}

In[477]:= ITDz2w[v_, w_, x_, y_, z_] := 0
```

```
In[478]:= ITDz3w[v_, w_, x_, y_, z_] := 6 / 2 !

In[479]:= ITDz4w[v_, w_, x_, y_, z_] := 24 z / 2 !

In[480]:= ITDz5w[v_, w_, x_, y_, z_] := (30 y + 60 z^2) / 2 !

In[481]:= ITDz6w[v_, w_, x_, y_, z_] := (36 x + 144 y z + 120 z^3) / 2 !

In[482]:= ITDz7w[v_, w_, x_, y_, z_] := (42 w + 84 y^2 + 168 x z + 420 y z^2 + 210 z^4) / 2 !

In[483]:= ITDz8w[v_, w_, x_, y_, z_] := (48 v + 192 x y + 192 w z + 480 y^2 z + 480 x z^2 + 960 y z^3 + 336 z^5) / 2 !

In[484]:= ITDz9w[v_, w_, x_, y_, z_] :=
  (108 x^2 + 180 y^3 + 216 v z + 972 y^2 z^2 + 810 y z^4 + 168 z^6 + 27 w (8 y + 12 z^2) +
    9 x (120 y z + 120 z^3) + z (216 w z + 648 y^2 z + 1080 y z^3 + 336 z^5)) / 2 !

In[485]:= ITDz10w[v_, w_, x_, y_, z_] :=
  (600 x y^2 + 600 x^2 z + 1200 y^3 z + 3600 x y z^2 + 4200 y^2 z^3 + 2100 x z^4 + 3360 y z^5 +
    720 z^7 + 10 v (24 y + 60 z^2) + 10 w (24 x + 120 y z + 120 z^3)) / 2 !

In[486]:= T3D = RecurrenceTable[
   {a[n] == FCS[[1]] * a[n - 1] + FCS[[2]] * a[n - 2] + FCS[[3]] * a[n - 3] + FCS[[4]] * a[n - 4] +
      FCS[[5]] * a[n - 5] + FCS[[6]] * a[n - 6] + FCS[[7]] * a[n - 7] +
      FCS[[8]] * a[n - 8] + FCS[[9]] * a[n - 9], a[0] == ITDz2w[0, 0, x, y, z] / 2,
    a[1] == ITDz3w[0, 0, x, y, z] / 3, a[2] == ITDz4w[0, 0, x, y, z] / 4,
    a[3] == ITDz5w[0, 0, x, y, z] / 5, a[4] == ITDz6w[0, 0, x, y, z] / 6,
    a[5] == ITDz7w[0, 0, x, y, z] / 7, a[6] == ITDz8w[0, 0, x, y, z] / 8,
    a[7] == ITDz9w[0, 0, x, y, z] / 9, a[8] == ITDz10w[0, 0, x, y, z] / 10}, a, {n, 0, 22}]

Out[486]= {0, 1, 3, 9, 25, 63, 153, 359, 819, 1830, 4018, 8694, 18 582, 39 298, 82 350,
  171 186, 353 338, 724 719, 1 478 061, 2 999 175, 6 057 687, 12 183 945, 24 411 935}
```

Verification of TD3

```
In[487]:= F3[n_] := Sum[FD3[[k]] * S3D[[n + 1 - k]], {k, 1, n}]

In[488]:= Table[F3[n], {n, 1, 20}]

Out[488]= {0, 1, 3, 9, 25, 63, 153, 359, 819, 1830, 4018, 8694,
  18 582, 39 298, 82 350, 171 186, 353 338, 724 719, 1 478 061, 2 999 175}
```

Fourth Derivative by convolution

```
In[489]:= F322[n_] := Sum[S3D[[k]] * S3D[[n + 1 - k]], {k, 1, n}]

In[490]:= Table[F322[n], {n, 1, 20}]

Out[490]= {1, 4, 14, 44, 125, 336, 864, 2144, 5174, 12 200, 28 212, 64 168,
  143 878, 318 608, 697 840, 1 513 664, 3 254 831, 6 944 396, 14 712 066, 30 968 980}
```

Fifth Derivative by Convolution

```
In[491]:= F323[n_] := Sum[S3D[[k]] * T3D[[n + 1 - k]], {k, 1, n}]
```

In[492]:=
```
Table[F323[n], {n, 1, 20}]
```
Out[492]=
```
{0, 1, 5, 20, 70, 220, 646, 1800, 4810, 12430, 31240, 76692, 184530,
 436340, 1016170, 2334880, 5301054, 11906935, 26487265, 58407320}
```

Second Order Parent Sequences for x^2-c1*x-c0 = 0 WH2 = (0, 0, 0, c0, c1)

In[493]:=
```
Clear[x, y, z]
```
In[494]:=
```
WH2 = {0, 0, 0, 2, 1}
```
Out[494]=
```
{0, 0, 0, 2, 1}
```
In[495]:=
```
y = WH2[[4]]
```
Out[495]=
```
2
```
In[496]:=
```
z = WH2[[5]]
```
Out[496]=
```
1
```

The Parent Sequence

In[497]:=
```
ISP1w[v_, w_, x_, y_, z_] := z
```
In[498]:=
```
ISP2w[v_, w_, x_, y_, z_] := 2 y + z^2
ISP3w[v_, w_, x_, y_, z_] := 3 x + 3 y z + z^3
```
In[500]:=
```
RecurrenceTable[{a[n] == WH2[[5]] * a[n - 1] + WH2[[4]] * a[n - 2],
  a[0] == ISP1w[0, 0, x, y, z], a[1] == ISP2w[0, 0, x, y, z]}, a, {n, 0, 22}]
```
Out[500]=
```
{1, 5, 7, 17, 31, 65, 127, 257, 511, 1025, 2047, 4097, 8191, 16385, 32767,
 65537, 131071, 262145, 524287, 1048577, 2097151, 4194305, 8388607}
```

The Element Sequence

In[501]:=
```
IDz1w[v_, w_, x_, y_, z_] := 1
```
In[502]:=
```
IDz2w[v_, w_, x_, y_, z_] := 2 z
```
In[503]:=
```
FD2 = RecurrenceTable[{a[n] == WH2[[5]] * a[n - 1] + WH2[[4]] * a[n - 2],
   a[0] == 0, a[1] == IDz1w[0, 0, x, y, z]}, a, {n, 0, 22}]
```
Out[503]=
```
{0, 1, 1, 3, 5, 11, 21, 43, 85, 171, 341, 683, 1365, 2731,
 5461, 10923, 21845, 43691, 87381, 174763, 349525, 699051, 1398101}
```

Second derivative- Convolution of Element sequence with itself

In[504]:=
```
CFS2 = {2 z, 2 y - z^2, -2 y * z, -y^2}
```
Out[504]=
```
{2, 3, -4, -4}
```
In[505]:=
```
ISDz2w[v_, w_, x_, y_, z_] := 2
```
In[506]:=
```
ISDz3w[v_, w_, x_, y_, z_] := 6 z
```
In[507]:=
```
ISDz4w[v_, w_, x_, y_, z_] := 8 y + 12 z^2
```
In[508]:=
```
ISDz5w[v_, w_, x_, y_, z_] := 10 x + 20 y z + 8 z^3 + z (10 y + 12 z^2)
```
In[509]:=
```
ISDz6w[v_, w_, x_, y_, z_] := 12 w + 18 y^2 + 36 x z + 72 y z^2 + 30 z^4
```

In[510]:=
```
ISDz7w[v_, w_, x_, y_, z_] := 14 v + 42 x y + 42 w z + 84 y^2 z + 84 x z^2 + 140 y z^3 + 42 z^5
```

In[511]:=
```
SD2 = RecurrenceTable[
  {a[n] == CFS2[[1]] * a[n - 1] + CFS2[[2]] * a[n - 2] + CFS2[[3]] * a[n - 3] + CFS2[[4]] * a[n - 4],
   a[0] == ISDz2w[0, 0, 0, y, z] / 2, a[1] == ISDz3w[0, 0, 0, y, z] / 3,
   a[2] == ISDz4w[0, 0, 0, y, z] / 4, a[3] == ISDz5w[0, 0, 0, y, z] / 5}, a, {n, 0, 22}]
```

Out[511]=
```
{1, 2, 7, 16, 41, 94, 219, 492, 1101, 2426, 5311, 11 528, 24 881, 53 398, 114 083,
 242 724, 514 581, 1 087 410, 2 291 335, 4 815 680, 10 097 401, 21 126 862, 44 117 867}
```

Third Derivative -Convolution of second derivative with element sequence

In[512]:=
```
TCS = {3 z, 3 (y - z^2), z^3 - 6 z * y, 3 * (z^2 - y) * y, 3 y^2 * z, y^3}
```

Out[512]=
```
{3, 3, -11, -6, 12, 8}
```

In[513]:=
```
ITDz2w[v_, w_, x_, y_, z_] := 0
```

In[514]:=
```
ITDz3w[v_, w_, x_, y_, z_] := 6 / 2!
```

In[515]:=
```
ITDz4w[v_, w_, x_, y_, z_] := 24 z / 2!
```

In[516]:=
```
ITDz5w[v_, w_, x_, y_, z_] := (30 y + 60 z^2) / 2!
```

In[517]:=
```
ITDz6w[v_, w_, x_, y_, z_] := (36 x + 144 y z + 120 z^3) / 2!
```

In[518]:=
```
ITDz7w[v_, w_, x_, y_, z_] := (42 w + 84 y^2 + 168 x z + 420 y z^2 + 210 z^4) / 2!
```

In[519]:=
```
ITDz8w[v_, w_, x_, y_, z_] := (48 v + 192 x y + 192 w z + 480 y^2 z + 480 x z^2 + 960 y z^3 + 336 z^5) / 2!
```

In[520]:=
```
ITDz9w[v_, w_, x_, y_, z_] :=
 (108 x^2 + 180 y^3 + 216 v z + 972 y^2 z^2 + 810 y z^4 + 168 z^6 + 27 w (8 y + 12 z^2) +
   9 x (120 y z + 120 z^3) + z (216 w z + 648 y^2 z + 1080 y z^3 + 336 z^5)) / 2!
```

In[521]:=
```
ITDz10w[v_, w_, x_, y_, z_] :=
 (600 x y^2 + 600 x^2 z + 1200 y^3 z + 3600 x y z^2 + 4200 y^2 z^3 + 2100 x z^4 + 3360 y z^5 +
   720 z^7 + 10 v (24 y + 60 z^2) + 10 w (24 x + 120 y z + 120 z^3)) / 2!
```

In[522]:=
```
TD2 = RecurrenceTable[{a[n] == TCS[[1]] * a[n - 1] + TCS[[2]] * a[n - 2] +
     TCS[[3]] * a[n - 3] + TCS[[4]] * a[n - 4] + TCS[[5]] * a[n - 5] + TCS[[6]] * a[n - 6],
   a[0] == ITDz2w[0, 0, 0, y, z] / 2, a[1] == ITDz3w[0, 0, 0, y, z] / 3,
   a[2] == ITDz4w[0, 0, 0, y, z] / 4, a[3] == ITDz5w[0, 0, 0, y, z] / 5,
   a[4] == ITDz6w[0, 0, 0, y, z] / 6, a[5] == ITDz7w[0, 0, 0, y, z] / 7}, a, {n, 0, 22}]
```

Out[522]=
```
{0, 1, 3, 12, 34, 99, 261, 678, 1692, 4149, 9959, 23 568, 55 014, 127 031, 290 457,
 658 602, 1 482 240, 3 314 025, 7 365 915, 16 285 300, 35 832 810, 78 500 811, 171 293 293}
```

Verification of TD2

In[523]:=
$$F2[n\_] := \sum_{k=1}^{n} FD2[[k]] * SD2[[n + 1 - k]]$$

In[524]:=
```
Table[F2[n], {n, 1, 20}]
```

Out[524]=
```
{0, 1, 3, 12, 34, 99, 261, 678, 1692, 4149, 9959, 23 568, 55 014,
 127 031, 290 457, 658 602, 1 482 240, 3 314 025, 7 365 915, 16 285 300}
```

Fourth Derivative by convolution

In[525]:=
```
F22[n_] := Sum[SD2[[k]] * SD2[[n + 1 - k]], {k, 1, n}]
```

$$F22[n\_] := \sum_{k=1}^{n} SD2[[k]] * SD2[[n+1-k]]$$

In[526]:=
```
Table[F22[n], {n, 1, 20}]
```

Out[526]= {1, 4, 18, 60, 195, 576, 1644, 4488, 11 925, 30 860, 78 278, 195 012, 478 599, 1 159 080, 2 774 880, 6 575 280, 15 439 065, 35 955 540, 83 118 970, 190 862 860}

Fifth Derivative by Convolution

In[527]:=

$$F23[n\_] := \sum_{k=1}^{n} SD2[[k]] * TD2[[n+1-k]]$$

In[528]:=
```
Table[F23[n], {n, 1, 20}]
```

Out[528]= {0, 1, 5, 25, 95, 340, 1106, 3430, 10 130, 28 915, 80 035, 216 143, 571 225, 1 482 110, 3 783 640, 9 522 740, 23 665 300, 58 149 845, 141 435 985, 340 854 645}

Proof Showing the Second Derivative ISP equals the Convolution of the First Derivative ISPs

In[529]:=
```
Clear[x, y, z]
```

In[530]:=
```
v = 0
```

Out[530]= 0

In[531]:=
```
w = 0
```

Out[531]= 0

First 7 terms of the first derivative of the ISP

In[532]:=

$$FD = \{1, (2 z)/2, (3 y + 3 z^2)/3, (4 x + 8 y z + 4 z^3)/4, (5 w + 5 y^2 + 10 x z + 5 y z^2 + z^4 + z (10 y z + 4 z^3))/5, (6 v + 12 w z + 18 y^2 z + 24 y z^3 + 6 z^5 + 6 x (2 y + 3 z^2))/6, (7 x^2 + 7 y^3 + 14 v z + 42 x y z + 42 y^2 z^2 + 28 x z^3 + 35 y z^4 + 7 z^6 + 7 w (2 y + 3 z^2))/7\}$$

Out[532]=

$$\left\{1, z, \frac{1}{3}(3 y + 3 z^2), \frac{1}{4}(4 x + 8 y z + 4 z^3), \frac{1}{5}(5 y^2 + 10 x z + 5 y z^2 + z^4 + z (10 y z + 4 z^3)), \frac{1}{6}(18 y^2 z + 24 y z^3 + 6 z^5 + 6 x (2 y + 3 z^2)), \frac{1}{7}(7 x^2 + 7 y^3 + 42 x y z + 42 y^2 z^2 + 28 x z^3 + 35 y z^4 + 7 z^6)\right\}$$

Eight term of the second derivative

In[533]:=

$$ISDz8 = (24 x^2 + 32 y^3 + 48 v z + 192 x y z + 240 y^2 z^2 + 160 x z^3 + 240 y z^4 + 56 z^6 + 8 w (6 y + 12 z^2))/8$$

Out[533]=

$$\frac{1}{8}(24 x^2 + 32 y^3 + 192 x y z + 240 y^2 z^2 + 160 x z^3 + 240 y z^4 + 56 z^6)$$

Recurrence Coefficients

In[534]:=
```
CFS = Reverse[{2 * z, 2 * y - z^2, 2 * (x - z * y), -2 x * z - y^2, -2 * x * y, -x^2}]
```

Out[534]=

$$\{-x^2, -2 x y, -y^2 - 2 x z, 2 (x - y z), 2 y - z^2, 2 z\}$$

Convolution of the first derivative ISPs

In[535]:= $F[n\_] := \sum_{k=1}^{n} FD[[k]] * FD[[n+1-k]]$

```
In[536]:= FS = Table[F[n], {n, 1, 6}]
```

Out[536]= $\left\{1,\ 2z,\ z^2 + \frac{2}{3}(3y + 3z^2),\ \frac{2}{3}z(3y + 3z^2) + \frac{1}{2}(4x + 8yz + 4z^3),\ \frac{1}{9}(3y + 3z^2)^2 + \frac{1}{2}z(4x + 8yz + 4z^3) + \frac{2}{5}(5y^2 + 10xz + 5yz^2 + z^4 + z(10yz + 4z^3)),\ \frac{1}{6}(3y + 3z^2)(4x + 8yz + 4z^3) + \frac{1}{3}(18y^2z + 24yz^3 + 6z^5 + 6x(2y + 3z^2)) + \frac{2}{5}z(5y^2 + 10xz + 5yz^2 + z^4 + z(10yz + 4z^3))\right\}$

Scalar product of the recurrence coefficients with the 1st 6 terms of the convolution

```
In[537]:= CFS.FS
```

Out[537]= $-x^2 - 4xyz + (-y^2 - 2xz)\left(z^2 + \frac{2}{3}(3y + 3z^2)\right) + 2(x - yz)\left(\frac{2}{3}z(3y + 3z^2) + \frac{1}{2}(4x + 8yz + 4z^3)\right) + (2y - z^2)\left(\frac{1}{9}(3y + 3z^2)^2 + \frac{1}{2}z(4x + 8yz + 4z^3) + \frac{2}{5}(5y^2 + 10xz + 5yz^2 + z^4 + z(10yz + 4z^3))\right) + 2z\left(\frac{1}{6}(3y + 3z^2)(4x + 8yz + 4z^3) + \frac{1}{3}(18y^2z + 24yz^3 + 6z^5 + 6x(2y + 3z^2)) + \frac{2}{5}z(5y^2 + 10xz + 5yz^2 + z^4 + z(10yz + 4z^3))\right)$

Simplify the output

In[538]:= **Simplify**$\left[-x^2 - 4xyz + (-y^2 - 2xz)\left(z^2 + \frac{2}{3}(3y + 3z^2)\right) + 2(x - yz)\left(\frac{2}{3}z(3y + 3z^2) + \frac{1}{2}(4x + 8yz + 4z^3)\right) + (2y - z^2)\left(\frac{1}{9}(3y + 3z^2)^2 + \frac{1}{2}z(4x + 8yz + 4z^3) + \frac{2}{5}(5y^2 + 10xz + 5yz^2 + z^4 + z(10yz + 4z^3))\right) + 2z\left(\frac{1}{6}(3y + 3z^2)(4x + 8yz + 4z^3) + \frac{1}{3}(18y^2z + 24yz^3 + 6z^5 + 6x(2y + 3z^2)) + \frac{2}{5}z(5y^2 + 10xz + 5yz^2 + z^4 + z(10yz + 4z^3))\right)\right]$

Out[538]= $3x^2 + 4y^3 + 30y^2z^2 + 30yz^4 + 7z^6 + 4x(6yz + 5z^3)$

Compared to the 8th ISP term of the second derivative shows it is the same second derivative of the ISP (ISDz8)

In[539]:= **Simplify**$\left[\frac{1}{8}(24x^2 + 32y^3 + 192xyz + 240y^2z^2 + 160xz^3 + 240yz^4 + 56z^6)\right]$

Out[539]= $3x^2 + 4y^3 + 30y^2z^2 + 30yz^4 + 7z^6 + 4x(6yz + 5z^3)$

RT 11/2019